THE 346 GAP-FREE PUZZLES ON 1 TO 9 SQUARES (UNVERIFIED)

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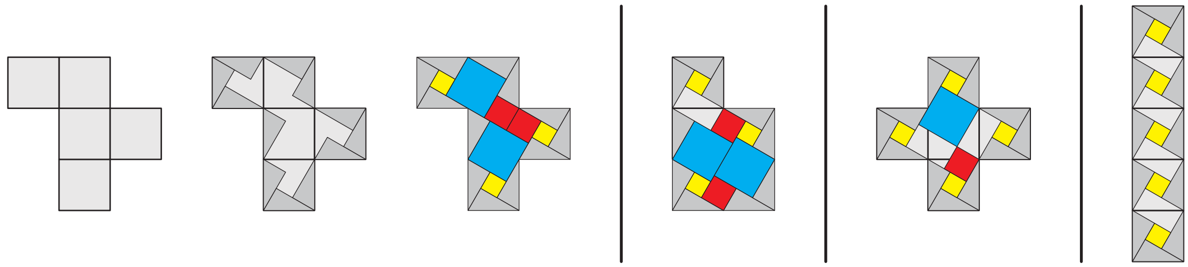
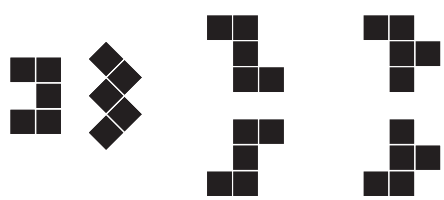
The figure shows a selection of Pythagapuzz puzzles using five base squares. We can completely fill the first with puzzle-piece squares; the others leave rectangular "gaps" the size of two triangles.



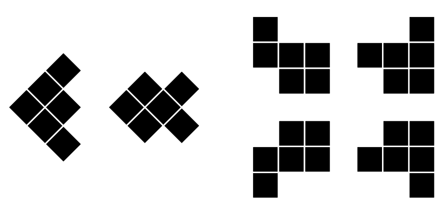
Categorizing all puzzles by number of gaps is an ambitious project beyond the scope of this note, which merely list all[1] "gap-free" puzzles on up to 9 squares. *Mirror-image* puzzles are included, since they have different —but related!— solutions, for readers who would like to use the list to keep track of their progress.

1, 2, 3, and 4 squares (1, 0, 1, and 1 gap-free puzzles)

5 Squares (6 gap-free puzzles)

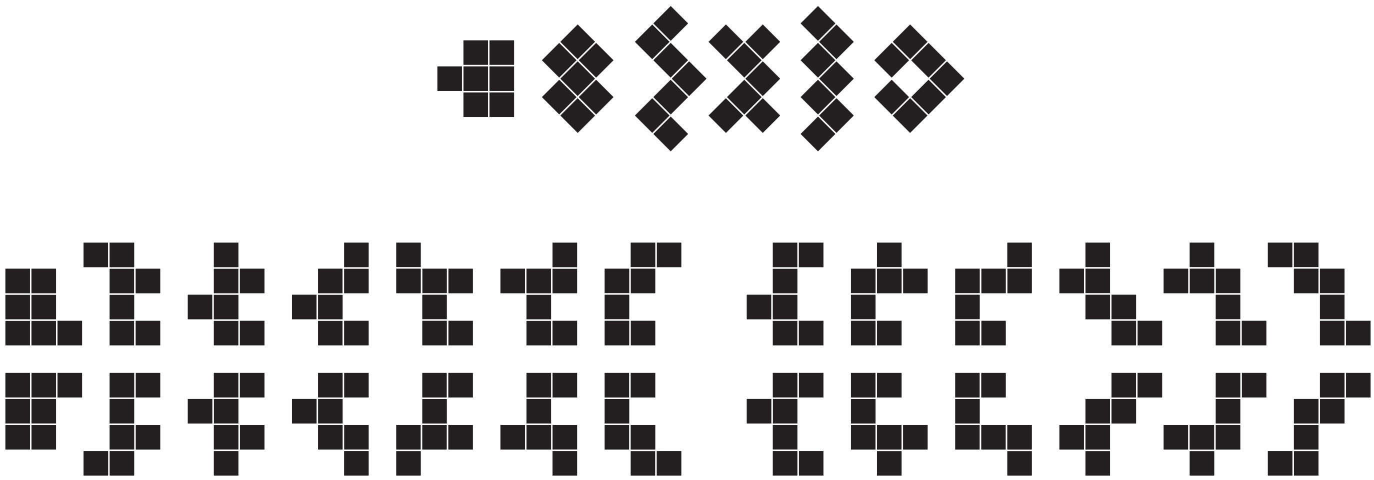


6 Squares (6 gap-free puzzles)

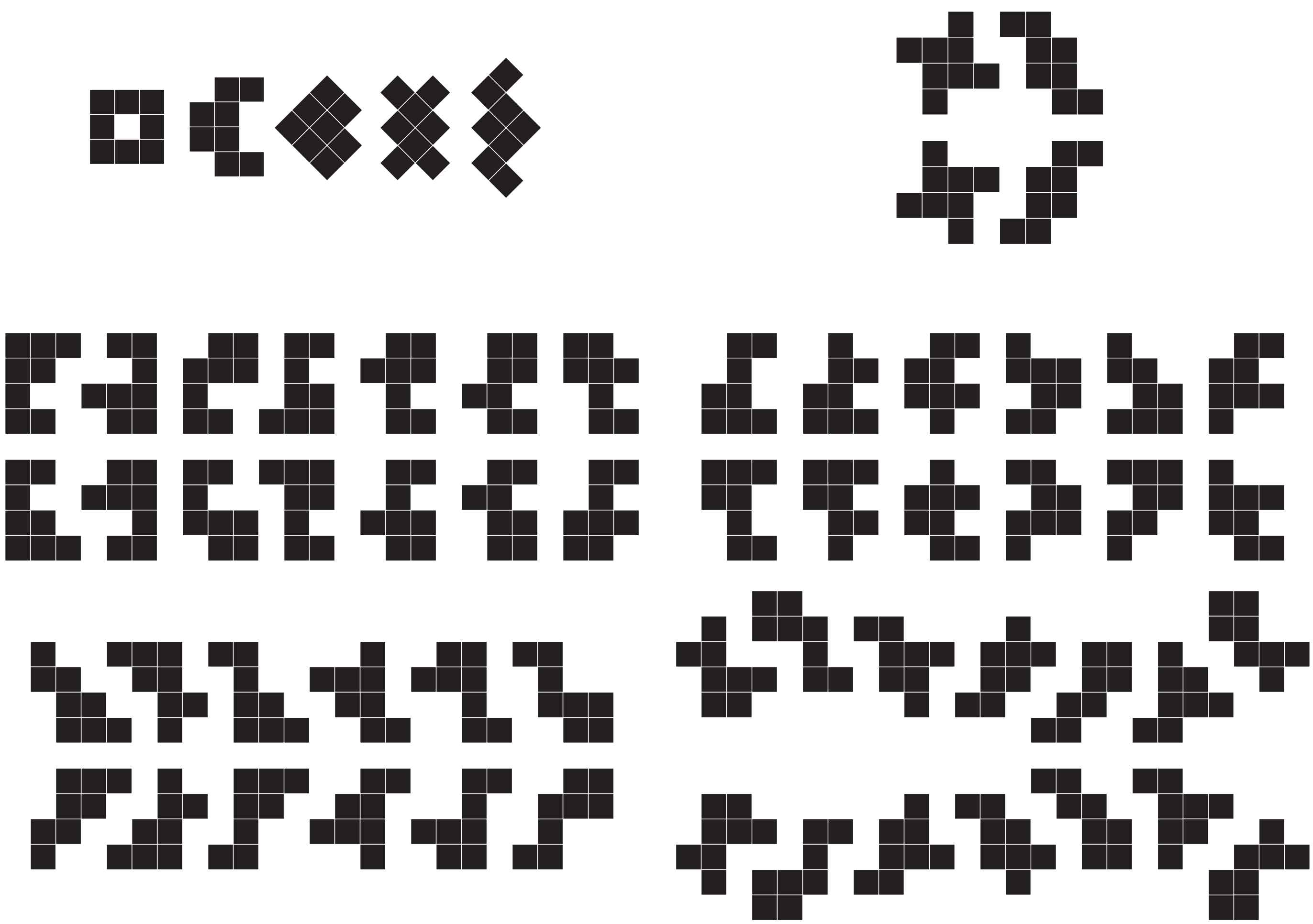


[1] This list was compiled manually(!). Please report any errors or omissions —or confirmations!— to `blue@pythagapuzz.com`

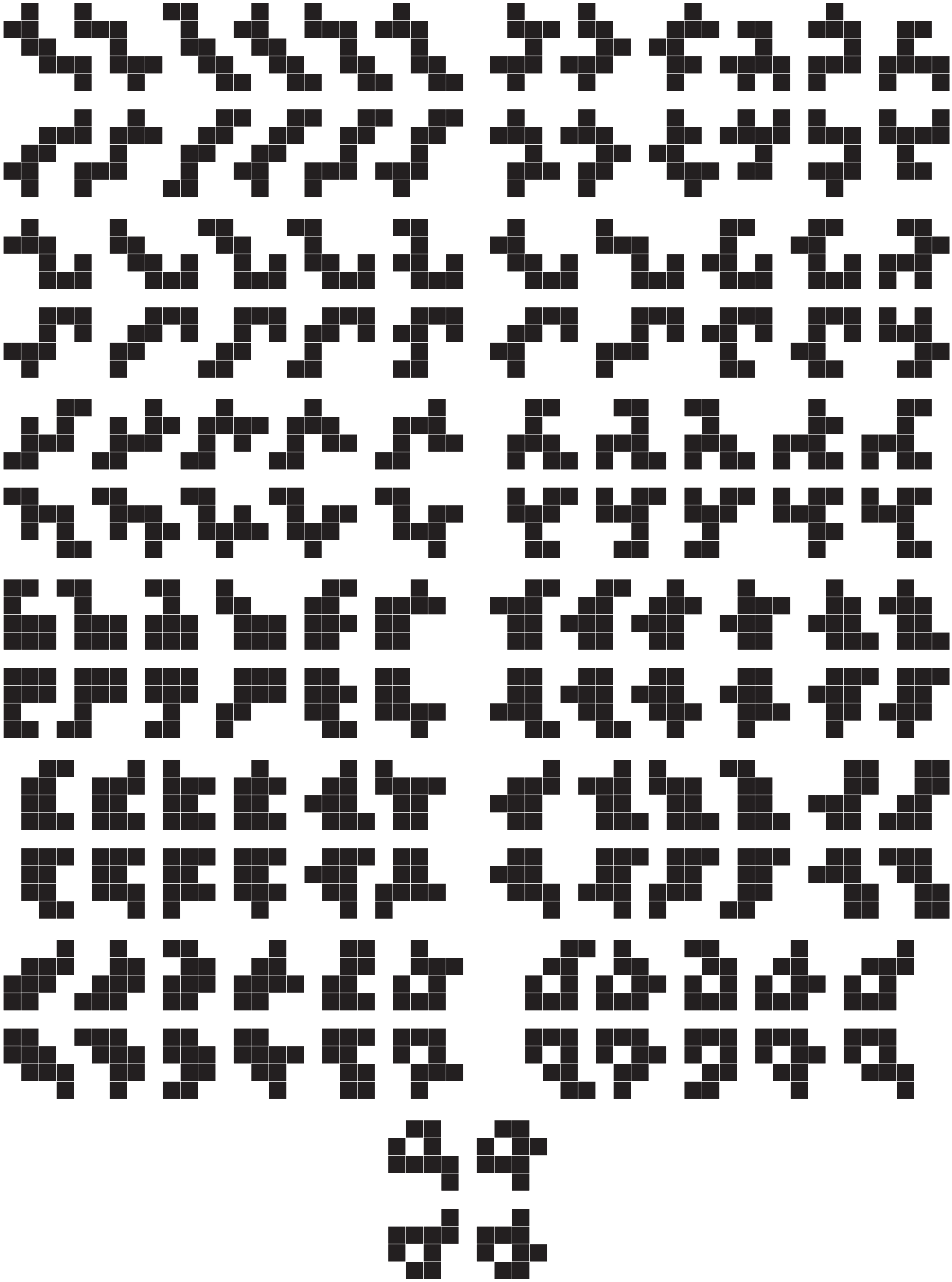
7 Squares (32 gap-free puzzles)



8 Squares (61 gap-free puzzles)



9 Squares (238 gap-free puzzles)



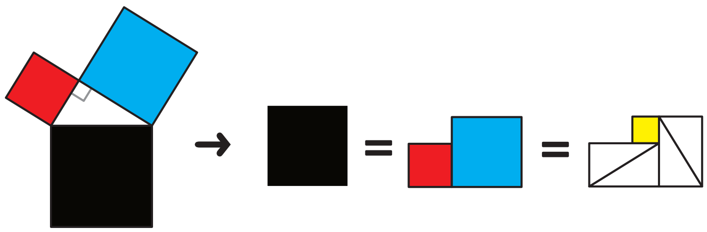
Appendix: Recognizing Gap-free Puzzles

> **Open Question:** How can we recognize Gap-free Puzzles ("GFP"s)?

While far from sufficient, the following rules describe some necessary properties of GFPs.

GFP Rule 1. The perimeter must be a multiple of 4.

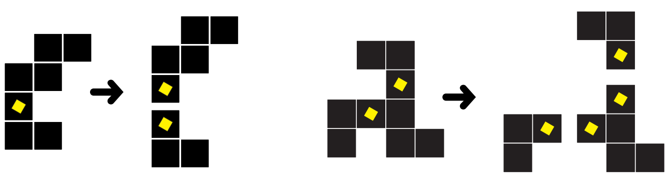
By the Pythagorean Theorem, a base square has area equal to the two larger puzzle-piece squares, which in turn are equal to the smallest square teamed with four triangles.



For a puzzle to be filled with squares alone —without any two-triangle gaps— the perimeter must account for all necessary triangles, hence must be a multiple of 4.

GFP Rule 2. Bridged sub-puzzles must be gap-free.

A "bridge" is a base square with neighbors along a pair of opposite edges, and without neighbors along the other pair of opposite edges. Removing all bridges from a puzzle separates remaining squares into two or more connected collections; a "sub-puzzle" is any one of those collections, along with all of the bridges attached to it.



A bridge gets filled by a smallest puzzle-piece square at its center. Consequently, any gaps in a puzzle occur within its sub-puzzles; and a lack-of-gaps in the puzzle requires a lack-of-gaps in each sub-puzzle.[2]

GFP Rule 2a. A bridge cannot neighbor a cap or another bridge.

A "cap" is a base square without neighbors along three edges. A bridge neighboring a cap or another bridge necessarily introduces a gap along the shared edge. (See, e.g., the rod-like puzzle at the far right of the first figure.)

> Even gapped puzzles —such as the "+"-shaped one in the first figure— may satisfy Rules 1 and 2 (the latter by virtue of having no bridges).

Rule 2a is perhaps the easiest filter to apply visually, since neighboring bridges or caps are readily noticed as snake-like portions of a puzzle. Rule 1 seems algorithm-friendly, requiring only a bit of simple counting.

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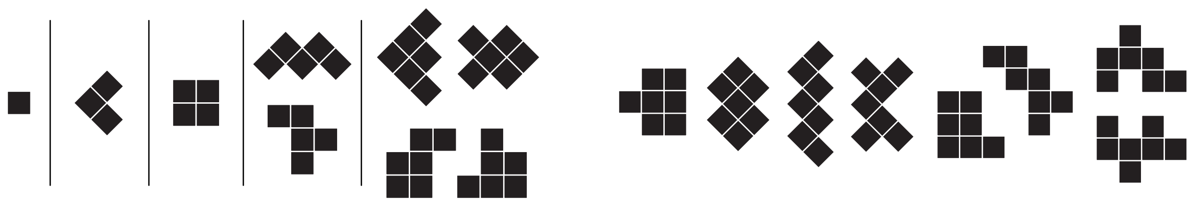
[2]We include the attached bridges in each sub-puzzle, as a gap could occur along the edge a bridge shares with a neighboring square.

Rule 2 is especially helpful, as it allows us to look at all the puzzles of a given size and tell which *can* or *cannot* be bridged sub-puzzles of larger puzzles.

Observe that a bridge-able sub-puzzle features one or more caps that overlap another's cap(s) to make the bridge(s). Thus, all bridged GFPs are generated by joining capped GFPs of smaller sizes; this suggests that the real work to be done in this area involves generating *bridge-free* —shall we say, *"prime"*?— GFPs. For quick reference, lists below cull the prime gap-free puzzles from the previous lists, omitting *mirror-image* versions for brevity:

1, 3, 4, 5, 6, and 7 Squares (1, 1, 1, 2, 4, and 8 prime GFPs)



8 and 9 Squares (22 and 44 prime GFPs)